

**FILMTEC Membranes****System Design: System Performance Projection****Design Equations  
and Parameters**

The performance of a specified RO system is defined by its feed pressure (or permeate flow, if the feed pressure is specified) and its salt passage. In its simplest terms, the permeate flow  $Q$  through an RO membrane is directly proportional to the wetted surface area  $S$  multiplied by the net driving pressure ( $\Delta P - \Delta \pi$ ). The proportionality constant is the membrane permeability coefficient or A-value. The familiar water permeation equation has the form:

$$Q = (A)(S)(\Delta P - \Delta \pi) \quad \text{Eq. 8}$$

The salt passage is by diffusion, hence the salt flux  $N_A$  is proportional to the salt concentration difference between both sides of the membrane. The proportionality constant is the salt diffusion coefficient or B-value.

$$N_A = B(C_{fc} - C_p) \quad \text{Eq. 9}$$

where:

$C_{fc}$  = feed-concentrate average concentration

$C_p$  = permeate concentration

There are basically two ways to calculate the performance of a specified design: "Element-to-Element" and "Entire System".

**Element-to-Element**

This is the most rigorous calculation method. It is too tedious for hand calculation, but it is suitable for computer calculations. All the operating conditions of the first element must be known, including the feed pressure. Then the flow, pressure, etc., of the concentrate, which is the feed to the second element, can be calculated. After calculating the results for all the elements, the original feed pressure may be too high or low, so the trial and error process starts with a new pressure.

With the help of the FILMTEC™ Reverse Osmosis System Analysis (ROSA) computer program, accurate results can be obtained very quickly, so this program can be used to modify and optimize the design of an RO or an NF system. Accordingly, the entire system calculation method will not be described here. It is also not intended to outline the process of the element to element computer calculation. However, the governing equations and parameters are given in Table 3.9.

In order to enable the determination of values for the terms  $A$ ,  $\Delta P$ , and  $\Delta \pi$  in Eq. 8, the water permeation equation is expanded to Eq. 10. The permeate concentration can be derived from Eq. 9 after conversion into Eq. 19. The design equations are listed in Table 3.9, the symbol definitions in Table 3.11.

The subscript  $i$  in the equations of Table 3.9 indicates that they apply to the  $i^{\text{th}}$  element in a sequence of  $n$  elements in a series flow configuration. To accurately determine system performance, Eq. 10 is successively solved for each of the  $n$  elements starting with an inlet set of conditions. The solutions depend on mass balances around each element for salt (Eq. 14) and water (Eq. 19), as well as correlations for individual element parameters such as concentrate-side flow resistance,  $\Delta P_{fc}$  (Eq. 27c); temperature correction factor for water permeability, TCF (Eq. 16); polarization factor,  $pf_i$  (Eq. 17), and the membrane permeability coefficient for water,  $A_i$  ( $\pi_i$ ) (Eq. 28) which in the case of the FILMTEC FT30 membrane depends on the average concentrate concentration or, alternatively, osmotic pressure. These solutions usually involve a suitable average for the feed and permeate side hydraulic and osmotic pressures. For low recovery values typical of single element operation, an accurate solution can be obtained using a simple arithmetic average of the inlet and outlet conditions. Even so, since the outlet conditions are not known, iterative trial and error solutions are involved.

**Table 3.9 Design equations for projecting RO system performance: individual element performance**

| Item                                                                             | Equation                                                                                                                                                                                                                                        | Equation Number |
|----------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------|
| Permeate flow                                                                    | $Q_i = A_i \bar{\pi}_i S_E (\text{TCF})(\text{FF}) \left( P_{fi} - \frac{\Delta P_{fc_i}}{2} - P_{pi} - \bar{\pi} + \pi_{pi} \right)$                                                                                                           | 10              |
| Average concentrate-side osmotic pressure                                        | $\bar{\pi}_i = \pi_{fi} \left( \frac{C_{fci}}{C_{fi}} \right) (pf_i)$                                                                                                                                                                           | 11              |
| Average permeate-side osmotic pressure                                           | $\bar{\pi}_{pi} = \pi_{fi} (1 - R_i)$                                                                                                                                                                                                           | 12              |
| Ratio: arithmetic average concentrate-side to feed concentration for Element $i$ | $\frac{C_{fci}}{C_{fi}} = \frac{1}{2} \left( 1 + \frac{C_{ci}}{C_{fi}} \right)$                                                                                                                                                                 | 13              |
| Ratio: concentrate to feed concentration for Element $i$                         | $\frac{C_{ci}}{C_{fi}} = \frac{1 - Y_i (1 - R_i)}{(1 - Y_i)}$                                                                                                                                                                                   | 14              |
| Feed water osmotic pressure                                                      | $\pi_f = 1.12(273 + T) \sum m_j$                                                                                                                                                                                                                | 15              |
| Temperature correction factor for RO and NF membrane                             | $\text{TCF} = \text{EXP} \left[ 2640 \left( \frac{1}{298} - \frac{1}{273 + T} \right) \right]; T \geq 25^\circ\text{C}$ $\text{TCF} = \text{EXP} \left[ 3020 \left( \frac{1}{298} - \frac{1}{273 + T} \right) \right]; T \leq 25^\circ\text{C}$ | 16a,b           |
| Concentration polarization factor for FILMTEC 8-inch elements                    | $pf_i = \text{EXP}[0.7Y_i]$                                                                                                                                                                                                                     | 17              |
| System recovery                                                                  | $Y = 1 - [(1 - Y_1)(1 - Y_2) \dots (1 - Y_n)] = 1 - \prod_{i=1}^n (1 - Y_i)$                                                                                                                                                                    | 18              |
| Permeate concentration                                                           | $C_{pi} = B(C_{fci})(pf_i)(\text{TCF}) \frac{S_E}{Q_i}$                                                                                                                                                                                         | 19              |

## Entire System

Average values are used to calculate feed pressure and permeate quality if the feed quality, temperature, permeate flow rate and number of elements are known. If the feed pressure is specified instead of the number of elements, the number of elements can be calculated with a few iterations. The design equations for 8-inch BW30 FILMTEC elements are listed in Table 3.10, the symbol definitions in Table 3.11.

**Table 3.10 Design equations for projecting RO system performance: system average performance**

| Item                                                                                     | Equation                                                                                                                                                                                                                                            | Equation Number |
|------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------|
| Total permeate flow                                                                      | $Q = N_E S_E \bar{A} \bar{\pi} (\text{TCF})(\text{FF}) P_f - \frac{\Delta P_{fc}}{2} P_p - \pi_f \left[ \frac{\bar{C}_{fc}}{C_f} p_f - (1 - \bar{R}) \right]$                                                                                       | 20              |
| Ratio: average concentrate-side to feed concentration for system                         | $\frac{C_{fc}}{C_f} = \frac{-\bar{R} \ln(1 - Y/Y_L)}{Y - (1 - Y_L) \ln(1 - Y/Y_L)} + (1 - \bar{R})$                                                                                                                                                 | 21              |
| Limiting system recovery                                                                 | $Y_L = 1 - \frac{\pi_f (\bar{p}f)(\bar{R})}{P_f - \Delta P_{fc} - P_p}$                                                                                                                                                                             | 22              |
| Approximate log-mean concentrate-side to feed concentration ratio for system             | $\left. \frac{C_{fc}}{C_f} \right _{Y_L, \bar{R}=1} = - \frac{\ln(1 - Y)}{Y}$                                                                                                                                                                       | 23              |
| Average element recovery                                                                 | $Y_i = 1 - (1 - Y)^{1/n}$                                                                                                                                                                                                                           | 24              |
| Average polarization factor                                                              | $\bar{p}f = \text{EXP}[0.7 \bar{Y}_i]$                                                                                                                                                                                                              | 25              |
| Average concentrate-side osmotic pressure for system                                     | $\bar{\pi} = \pi_i \left( \frac{\bar{C}_{fc}}{C_f} \right) \bar{p}f$                                                                                                                                                                                | 26              |
| Average concentrate-side system pressure drop for FILMTEC 8-inch elements; 2 stages      | $\Delta P_{fc} = 0.04 \bar{q}_{fc}^2$<br>$\Delta P_{fc} = \left[ \frac{0.1(Q/1440)}{Y N_{V2}} \right] \left( \frac{1}{N_{VR}} + 1 - Y \right)$                                                                                                      | 27a,b,c         |
| Individual FILMTEC 8-inch element, or single-stage concentrate-side pressure drop        | $\Delta P_{fc} = 0.01 n \bar{q}_{fc}^{1.7}$                                                                                                                                                                                                         |                 |
| FILMTEC membrane permeability as a function of average concentrate-side osmotic pressure | $\bar{A}(\bar{\pi}) = 0.125; \bar{\pi} \leq 25$<br>$\bar{A}(\bar{\pi}) = 0.125 - 0.011 \left( \frac{\bar{\pi} - 25}{35} \right); 25 \leq \bar{\pi} \leq 200$<br>$\bar{A}(\bar{\pi}) = 0.070 - 0.0001(\bar{\pi} - 200); 200 \leq \bar{\pi} \leq 400$ | 28a,b,c         |
| Permeate concentration                                                                   | $C_p = B C_{fc} \bar{p}f (\text{TCF}) \left( \frac{N_E S_E}{Q} \right)$                                                                                                                                                                             | 29              |

# Design Equations and Parameters

**Table 3.11 Symbol definitions**

(cont.)

|                   |                                                                                                                      |                       |                                                                                                              |
|-------------------|----------------------------------------------------------------------------------------------------------------------|-----------------------|--------------------------------------------------------------------------------------------------------------|
| $Q_i$             | permeate flow of Element $i$ (gpd)                                                                                   | $\sum_i$              | summation of all ionic species                                                                               |
| $A_i \pi_i$       | membrane permeability at 25° for Element $i$ , a function of the average concentrate-side osmotic pressure (gfd/psi) | $Y$                   | system recovery (expressed as a fraction) = permeate flow/feed flow                                          |
| $S_E$             | membrane surface area per element (ft <sup>2</sup> )                                                                 | $\prod_{i=1}^n$       | multiplication of $n$ terms in a series                                                                      |
| TCF               | temperature correction factor for membrane permeability                                                              | $n$                   | number of elements in series                                                                                 |
| FF                | membrane fouling factor                                                                                              | $Q$                   | system permeate flow (gpd)                                                                                   |
| $P_{fi}$          | feed pressure of Element $i$ (psi)                                                                                   | $N_E$                 | number of elements in system                                                                                 |
| $\Delta P_{fc i}$ | concentrate-side pressure drop for Element $i$ (psi)                                                                 | $\bar{Q}_i$           | average element permeate flow (gpd) = $Q/N_E$                                                                |
| $P_{pi}$          | permeate pressure of Element $i$ (psi)                                                                               | $\bar{A} \pi$         | average membrane permeability at 25°C: a function of the average concentrate-side osmotic pressure (gfd/psi) |
| $\bar{\pi}_i$     | average concentrate-side osmotic pressure (psi)                                                                      | $\bar{C}_{fc}$        | average concentrate-side concentration for system (ppm)                                                      |
| $\pi_{fi}$        | feed osmotic pressure of Element $i$                                                                                 | $\bar{R}$             | average fractional salt rejection for system                                                                 |
| $\pi_{pi}$        | permeate-side osmotic pressure of Element $i$ (psi)                                                                  | $\bar{\pi}$           | average concentrate-side osmotic pressure for system (psi)                                                   |
| $pf_i$            | concentration polarization factor for Element $i$                                                                    | $\Delta \bar{P}_{fc}$ | average concentrate-side system pressure drop (psi)                                                          |
| $R_i$             | salt rejection fraction for Element $i$<br>= $\frac{\text{feed conc.} - \text{perm. conc.}}{\text{feed conc.}}$      | $Y_L$                 | limiting (maximum) system recovery (expressed as a fraction)                                                 |
| $C_{fc i}$        | average concentrate-side concentration for Element $i$ (ppm)                                                         | $\bar{Y}_i$           | average element recovery (expressed as a fraction)                                                           |
| $C_{fi}$          | feed concentration for Element $i$ (ppm)                                                                             | $\bar{pf}$            | average concentration polarization factor                                                                    |
| $C_{ci}$          | concentrate concentration for Element $i$ (ppm)                                                                      | $\bar{q}_{fc}$        | arithmetic average concentrate-side flow rate (gpm)<br>(=1/2(feed flow + concentrate flow))                  |
| $Y_i$             | recovery fraction for Element $i$<br>= $\frac{\text{permeate flow}}{\text{feed flow}}$                               | $N_V$                 | number of six-element pressure vessels in system ( $\approx N_E/6$ )                                         |
| $\pi_f$           | treated feed water osmotic pressure (psi)                                                                            | $N_{V1}$              | number of pressure vessels in first stage of 2-stage system ( $\approx 1/3 N_V$ )                            |
| $T$               | feed water temperature (°C)                                                                                          | $N_{V2}$              | number of pressure vessels in second stage of 2-stage system ( $\approx N_V/3$ )                             |
| $m_j$             | molal concentration of $j^{\text{th}}$ ion species                                                                   | $N_{VR}$              | stage ratio (= $N_{V1}/N_{V2}$ )                                                                             |

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